Chapter Five
Sensible Synthesis

Central to Kant's doctrine in the *Critique of Pure Reason* is the distinction between spontaneity and receptivity. This distinction is tied to a distinction between the intellect and sensibility. The intellect, or understanding, is spontaneous, while sensibility is receptive. The understanding is characterized by Kant as a capacity for judgment, and a wide-spread view among commentators takes this claim to entail that to exercise the spontaneous capacity of the mind is to judge. I argued in Chapter Three that this view is mistaken. There are good reasons for thinking that Kant recognizes a kind of exercise of spontaneity that is distinct from judgment; what I have been calling its exercise in sensible synthesis. The task of this chapter is twofold. First, I want to give a more precise characterization of the exercise of spontaneity in sensible synthesis. More specifically, I want to say more about why this act must be regarded as sensible rather than discursive. My claim is that the sensible exercise of spontaneity is modeled on mathematical construction and that reflection on Kant's conception of mathematical construction shows that, and why, such an act cannot be regarded as an act of judgment, but instead must be regarded as an act of specifically sensible synthesis. The second task of the chapter is to establish that sensible synthesis can be understood as a kind of apperceptive synthesis. In Chapter Four I argued that the understanding is most fundamentally a capacity for apperceptive synthesis. My claim was that this characterization of the understanding is fundamental because it allows us to comprehend the unity of this capacity; that is, it allows us to comprehend that judgment and sensible synthesis, though distinct in character, are nevertheless acts of the same capacity. To establish this claim it must be shown, first, that judgment is indeed a kind of apperceptive synthesis and, second, that sensible synthesis is indeed a kind of apperceptive synthesis. I made the case for the first claim in the preceding chapter. The task now is to make the parallel case for sensible synthesis.

I shall begin by briefly rehearsing the reasons why there must be a sensible exercise of spontaneity (§1). Then I shall try to bring out the central differences between discursive representation and sensible representation (that is, representation through concepts and representation through intuition) by discussing Kant's conception of mathematical cognition. Mathematics, according to Kant, proceeds by constructing concepts in pure intuition. I argue that the mathematical notion of constructing a concept in pure intuition constitutes the paradigm case of the sensible exercise of spontaneity. In support of this claim, I give an exposition of Kant's conception of mathematical construction and show that construction is an act of spontaneous, yet specifically sensible synthesis (§2). I go on to show that Kant holds that every intuition, including empirical intuition, involves an act of the type paradigmatically exhibited in mathematical construction; hence that intuition quite generally depends on an exercise of spontaneity in sensible synthesis (§3) Finally, I argue that Kant's conception of spontaneity has room for a non-discursive exercise of this capacity by showing that sensible synthesis must be construed as the specifically sensible exercise of the capacity for apperceptive synthesis. It thus constitutes a species of a genus whose other species is the discursive exercise of the capacity for apperceptive synthesis, which is judgment (§4).

1. The Need for a Non-Discursive Act of Spontaneity

Intuition is defined by Kant as the singular, immediate representation of an object (cf. A320/B376f).[1] Through intuition, Kant says, objects are given to us, while through the understanding they are thought (A19/B33). For an object to be so given, and thus for an intuition to occur, the object must affect the mind (cf. ibid.). As I argued in Chapter Two I take this to mean, first, that intuition is object-dependent: Intuition depends on the sensory presence of the object it is of. Absent the object there can be no intuition of it. Second, intuition is of the particular, not of the general. Since intuition is spatio-temporal we can put this by saying that intuition is of what is here and now. It follows from this that intuition, just as such, is neither the representation of what kind of thing something is nor the representation of something's being an instance of a law.

More importantly, it follows from these characteristics that intuition, as such and by itself, cannot account for the representation of an object. Being a merely receptive representation, the content of an intuition is determined by what is affecting the senses. It is an array of sensory qualities, perceived from a certain perspective at a certain time. An object, however, is essentially something that outstrips such a perspectival representation. The idea of an object is the idea of a bearer of sensory qualities, which persists through changes of these qualities; which exists unperceived; which can be perceived from a variety of different perspectives that are systematically related to one another; and whose behavior is governed by universal laws.

Moreover, the idea of an object is the idea of something that supports modal considerations. Thought about objects is, in principle, thought about what something would be or do under such-and-such conditions. But intuition, in being object-dependent in the sense

[1] See Chapter One for discussion.

indicated, is essentially tied to what is actual, what is here and now. Therefore, the representation of an object cannot be just an act of intuition.

However, I said above that Kant defines intuition as the singular, immediate representation *of an object*. If what I just said is right, this must mean that intuition is something for which the capacity to have intuitions, sensibility, on its own cannot account. An intuition can be the representation of an object only through the collaboration of a non-receptive capacity. For Kant, of course, this is the understanding, the faculty of spontaneity. The understanding is the source of what Kant calls the concept of an object in general. This concept is articulated by the categories, or pure concepts of the understanding, and to say that these concepts are pure is to say that they, and thus the concept of an object in general, have their source in the understanding itself.[2] This means, first, that an intuition is the representation of an object only if it instantiates the concept of an object in general. And second, because the concept of an object in general is a pure concept, a concept that derives from the understanding alone, for intuition to instantiate this concept requires an act of the understanding.

This last point is important. It is of course a consequence of the claim that receptivity on its own cannot account for the object-representingness of intuition. But it is important to make this consequence explicit: An intuition is the representation of an object only by means of an act of the understanding, the faculty of spontaneity.

Kant prominently characterizes the understanding as, among other things, a capacity to judge. This has led a number of influential commentators to suggest that judgment is the act of the understanding on account of which intuition is of objects. More specifically, the view is that an intuition, which *qua* merely sensible representation is not object-related, becomes the

[2] For an elaboration of this point see Chapter Two, §3.

representation of an object through an act of judgment. Henry Allison's elaboration of this view is representative.

Because Allison recognizes that Kant's conception of intuition as the singular, immediate representation of an object combines two apparently incompatible demands – on the one hand, being a representation of sensibility and, on the other hand, being the representation of an object – he suggests that we need to draw a distinction between two kinds of intuition in Kant. Allison calls these unconceptualized (or indeterminate) intuitions and conceptualized (or determinate) intuitions, respectively.[3] The former are accounted for by the workings of sensibility alone. They are immediate but they do not constitute the representation of an object. Allison calls them "raw-data intuitions." By contrast, a conceptualized intuition is the representation of an object. But it cannot be accounted for by sensibility alone. As its name suggests, it depends on an act of the understanding, the power of concepts. More specifically, it depends on an act of judgment. For Allison envisions a conceptualized intuition as being the immediate referent of the subject-term in a categorical judgment. Thus, Allison takes the following formula to be the model of categorical judgment in Kant: 'The x (or x's) that I think through S I also think through P,' where 'x' stands for an intuition, while 'S' and 'P' stand for the subject- and predicate-term, respectively, of the judgment.[4]

In Chapter Three I argued that this conception is mistaken. Let me give a brief summary of what I take to be the main problem. We can get the issue into focus if we reflect on a fundamental difference between intuitions and concepts that came up in Chapter One: While an

[3] "[…] it is necessary to distinguish between a determinate or conceptualized and an indeterminate intuition, only the former of which constitutes a *repraesentatio singularis*" (Allison, *Kant's Transcendental Idealism*, 82). I should add that Allison is not alone in calling for such a distinction. A distinction along very similar lines is defended by Beck, "Did the Sage of Königsberg Have No Dreams?"
[4] Cf. Allison, *Kant's Transcendental Idealism*, 85-87.

intuition is the representation of an object as completely determinate, conceptual representation (i.e., judgment) is necessarily less than fully determinate.[5] For Kant this is a consequence of the generality of concepts and, correspondingly, of the singularity of intuition. A concept represents objects through marks (that is, general characteristics), which are in principle applicable to an indefinite number of objects. Examples are ‘being impenetrable,’ ‘being a body,’ etc. Such a mark, or general characteristic, is always further determinable. For instance, a body might be animate or inanimate. And an animate body might be a plant or an animal. And so on. For Kant the intension of a concept is a collection of marks. He takes it to follow from this fact that, no matter how many marks I include in my concept, it is always possible to introduce further differentiations along the lines indicated in the example. I can always, in principle, determine a given concept *C* further by finding a predicate *F* with regard to which I divide *C*’s extension into two sub-species, one comprising those *C*s which are *F*, the other comprising those *C*s which are not *F*.[6] I can then think of each of these subspecies as further determinations of my original concept, *C*. Kant’s point is that this process of determination can be continued indefinitely. At no stage is there a principle available which would rule out the possibility of carrying the process one step further.[7]

This is the upshot of Kant’s principle that there is no *infima species,* that is, no concept which is such that what falls under it differs only numerically but not qualitatively.[8] By contrast,

[5] See the discussion of Kant’s view that there is no *infima species* in Chapter One, §2.1.
[6] Recall that the extension of a concept for Kant comprises both the concepts that are subordinated to it and the objects that fall under it. Cf. Chapter One, §3.2.
[7] This does not mean that it is always expedient in practice to carry on the process of determination as far as possible. The point here is one of principle. As Kant says in the *Logik*, it is perfectly legitimate in practice to treat a concept as a lowest species, i.e. to treat it *as if* no further determination was possible. See Chapter One, §2.1, for references and discussion.
[8] At least, no concept whose content is fixed exclusively by conceptual marks. What Kant calls pure sensible concepts are an exception. Thus, the concept ‘region of space,’ for instance, is such that what falls under it differs

intuition is not determinable in this manner. An intuition, precisely because it is singular, presents its object as fully determinate, where this means that for every possible predicate there is a fact of the matter as to whether it or its contradictory opposite applies to the object. In contrast to concepts an intuitive representation itself is such that it leaves no room for further determination.[9]

However, the complete determinacy of intuition is sensible, not discursive. This means that the object's determinations are not represented *as* determinations. Their representation, that is, does not involve the consciousness of their generality, the consciousness that these determinations may also be instantiated by an indefinite number of other objects. As we might put it, what is represented in intuition is the fully determinate object bearing its determinations. But the determinations are not represented as determinations. As a consequence, what these determinations are still needs to be articulated. And to do so, that is, to separate them out and represent them as general properties, is the job of applying (material) concepts in judgment. Articulating the content of an intuition in this way may also be referred to as determining the intuition.[10] But it is important to note that this is a different sense of 'determine' from the one discussed just a moment ago in connection with the indefinite determinability of concepts.

With this characterization of the relation between intuition and concepts in mind I now turn to a brief discussion of the special character of the pure concepts of the understanding, or categories. This will allow me to bring out what is mistaken about Allison's picture and conclude

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only numerically but not qualitatively. However, the content of these concepts is not fixed by conceptual marks but in a crucial way depends on intuition. As long as we are limited to conceptual determination, that is, determination by means of logical division, we can never arrive at a concept that is maximally determinate.

[9] Determination in the sense just elaborated, that is. As I shall explain shortly, the term is also used in a different sense.

[10] I think this sense of 'determine' is at issue when Kant characterizes an appearance as "the undetermined object of an empirical intuition" (A20/B34).

this summary of my argument in favor of a non-discursive exercise of spontaneity. A pure concept of the understanding, Kant says, represents its object as being determined with regard to one of the logical forms of judgment (cf. B128). This means that a pure concept characterizes its object, not as falling under this or that concept, but as being a possible content of judgment in the first place, as being the kind of thing that can fall under concepts. A pure concept, we might say, does not classify its object as being of this rather than that sort. This is what material concepts do. By contrast, a pure concept characterizes its object merely as something to which material concepts are applicable. The function of pure concepts, therefore, is not to articulate the contents of intuitions. On the contrary, it is to represent intuitions as being susceptible to articulation in judgment, through the application of material concepts. Therefore, a pure concept is a formal concept.[11]

In light of these considerations we can now describe the problem with Allison's view as follows. I said above that an intuition is the representation of an object only if it exhibits the unity that is thought in the concept of an object in general. Since the categories spell out the content of this concept this amounts to saying that for an intuition to be the representation of an object is for its object to instantiate the categories. Thus far, Allison would agree. But as we have just seen, for the object of an intuition to instantiate the categories cannot amount to an articulation of the content of this intuition in judgment, as Allison would have it. To articulate the content of an intuition is to apply material concepts. However, to say that an object given in intuition instantiates the concept of an object in general is not to predicate material concepts of this object. It is rather to say that material concepts are applicable. It is to say that what is given in intuition is the kind of thing that can function as a subject of predication.

[11] Cf. the discussion of the notion of a pure concept in Chapter Two, §2.

If this is right, it shows that Allison's solution to the problem of accounting for the object-representingness of intuition fails. It confuses the articulation of the content of an intuition with its object-representingness. It confuses, we might say, the act of judgment and its role in cognition with what is supposed to make this act possible in the first place.

Using the terminology I introduced in Chapter Two we can put the point as follows. For an object given in intuition to instantiate the categories is to say that the intuition through which the object is given exhibits sensible modes of combination. By contrast, a judgment is characterized by exhibiting modes of concept-combination. But these are distinct modes of combination. Therefore, it is not the case that for an object given in intuition to instantiate the categories is *ipso facto* for it to be caught up in an act of judgment.

From a slightly different angle, we can also put the problem with Allison's position as follows. Kant defines intuition as the singular, immediate representation of an object. In Allison's picture these two characteristics come apart and get distributed across different kinds of representations. Thus, Allison's unconceptualized intuitions (the "raw data") are immediate but they are not representations of objects. By contrast, Allison's conceptualized intuitions are representations of objects but, being dependent on an act of judgment, they are no longer immediate.

If all of this is right, it becomes clear that the way in which spontaneity must be involved in intuition for it to be object-representing is not by means of acts of judgment. But if spontaneity must nonetheless be involved, then there must be a kind of exercise of spontaneity which does not take the form of judgment. There must be, as I call it, a non-discursive exercise of spontaneity. This kind of exercise of spontaneity must be an act that leaves the sensible character of intuition intact. It must be an act which, rather than replacing intuition with discursive representation or making the former a mere part, or aspect, of the latter, preserves the sensible character of intuition. Adapting a term of Kant's, I call this the act of sensible synthesis.

2. Geometrical Representation

2.1. The Object of Mathematical Cognition

I have argued that intuition can be what Kant says it is, viz. the immediate representation of an object, only if it exhibits the unity articulated by the pure concepts of the understanding, or categories. Since this requires an act of the spontaneous capacity of the mind it follows that there is an act of spontaneity which pertains directly to intuition. This cannot be a discursive act; that is, it cannot be an act of judgment. Part of the task of this chapter is to articulate the nature of this act. This requires, in the first instance, that we get a firmer grip on exactly how discursive representation differs from intuitive representation. In this section, I shall work towards this goal by discussing Kant's view of mathematical construction as it is used, paradigmatically, in geometry.[12]

As we shall see towards the end of this chapter, geometrical construction is the paradigmatic act of sensible synthesis for Kant. If we want to understand his conception of sensible synthesis we must therefore focus on his account of geometrical construction. This will

[12] A complete account of Kant's conception of mathematical construction would have to discuss his views on the use of construction in arithmetic and algebra. But for our purposes this is not necessary. It is clear that for Kant construction in geometry is the paradigm of mathematical construction in general. For discussion see Shabel, *Mathematics in Kant's Critical Philosophy*, 115-131, and Shabel, "Kant on the 'Symbolic Construction' of Mathematical Concepts."

require some background. I shall begin, therefore, with an account of Kant's conception of mathematical cognition in general.[13]

It will be best to begin with the distinction between philosophical cognition and mathematical cognition, as Kant sketches it in the chapter on the Discipline of Pure Reason in the Transcendental Doctrine of Method.[14] Philosophy and mathematics, he says, form the two species of the genus 'rational cognition' (*Vernunfterkenntnis*). The genus is characterized by its independence from experience: rational cognition is a priori cognition. It is cognition through reason alone, independent of experience. Its two species are characterized, respectively, as 'rational cognition from concepts' and 'rational cognition from the construction of concepts,' the former being philosophical cognition, the latter mathematical cognition.[15]

As Kant emphasizes, this distinction is drawn in terms of method. Thus, cognizing something 'from concepts,' and cognizing something 'from the construction of concepts' denote

[13] Since Kant's views on mathematics are not my primary focus here, I shall give only a summary presentation. My presentation is indebted to recent work by Emily Carson, Michael Friedman, Lisa Shabel, and Daniel Sutherland, which has substantially deepened our understanding of Kant's philosophy of mathematics and, in particular, of the role of intuition in it. Friedman has emphasized the role that the expressive limitations of traditional Aristotelian logic play in motivating Kant's view that mathematics depends on intuition (see Friedman, *Kant and the Exact Sciences*). However, in Friedman's work this emphasis on logic has led to a failure to appreciate what one might call the epistemic role of intuition in mathematical cognition, that is, the fact that intuition also serves to confer objective validity on mathematical concepts. This has been pointed out by Carson (see Carson, "Kant on Intuition in Geometry"; see also the modified position Friedman adopts in response to Carson's criticism in his "Geometry, Construction, and Intuition in Kant and His Successors"). Building on both Friedman and Carson, Sutherland has developed a sophisticated account of the intuition-dependence of mathematics which stresses the limitations of conceptual representation as Kant conceives it, but is at the same time sensitive to the epistemic role of intuition in mathematical cognition (see Sutherland, "Kant's Philosophy of Mathematics and the Greek Mathematical Tradition"; Sutherland, "The Role of Magnitude in Kant's Critical Philosophy"; and Sutherland, "Kant on Arithmetic, Algebra, and the Theory of Proportions"). A detailed account of Kant's conception of geometrical construction has been developed by Shabel (see Shabel, *Mathematics in Kant's Critical Philosophy*; Shabel, "Kant's Philosophy of Mathematics"; Shabel, "Reflections on Kant's Concept (and Intuition) of Space"; Shabel, "Kant on the 'Symbolic Construction' of Mathematical Concepts"). The account of geometrical construction I am about to sketch is greatly indebted to Shabel's work.
[14] Cf. A712/B740ff.
[15] "Philosophical cognition is rational cognition from concepts, mathematical cognition that from the construction of concepts" (Die philosophische Erkenntnis ist die Vernunfterkenntnis aus Begriffen, die mathematische aus der Konstruktion der Begriffe.) (A713/B741).

two distinct methods of inquiry. What these methods are, and how they differ, will concern us in a moment. For now what matters is the status Kant accords to them in drawing the distinction between the two sciences by reference to their methods. Kant is explicit that the reason why the two sciences are defined in terms of their methods is that the difference in method is prior to the difference in subject-matter, or object.[16] What these sciences are about is thus a function of how they proceed. Philosophy is about whatever can be cognized a priori from concepts. Mathematics is about whatever can be cognized a priori from the construction of concepts.[17]

For reasons which will become apparent throughout the course of this chapter, Kant holds that what can be cognized through the construction of concepts is magnitudes. Mathematical knowledge, therefore, is knowledge of magnitudes. Because all and only the concepts of magnitudes can be constructed, all and only mathematical cognition is of magnitudes. In a move that will appear strange to the modern reader, however, Kant draws a distinction between two different notions of magnitude. These are called, by their Latin names, *quantum* and *quantitas,* respectively. Of these, the first is the one that will appear foreign to us. As Daniel Sutherland, who has recently emphasized the importance of this distinction, explains it, a *quantum* is a

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[16] "The essential difference between these two kinds of rational cognition therefore consists in this form, and does not rest on the difference in their matter, or objects. Those who thought they could distinguish philosophy from mathematics by saying of the former that it takes as its object merely *quality*, while the latter takes *quantity*, took the effect for the cause. The form of mathematical cognition is the cause of its pertaining solely to quanta. For only the concept of magnitudes can be constructed, i.e., exhibited a priori in intuition, while qualities cannot be exhibited in anything but empirical intuition. Hence a rational cognition of the latter can be possible only through concepts." (In dieser Form besteht also der wesentliche Unterschied dieser beiden Arten der Vernunfterkenntnis, und beruhet nicht auf dem Unterschiede ihrer Materie, oder Gegenstände. Diejenigen, welche Philosophie von Mathematik dadurch zu unterscheiden vermeineten, daß sie von jener sagten, sie habe bloß die *Qualität*, diese aber nur die *Quantität* zum Objekt, haben die Wirkung für die Ursache genommen. Die Form der mathematischen Erkenntnis ist die Ursache, daß diese lediglich auf Quanta gehen kann. Denn nur der Begriff von Größen läßt sich konstruieren, d.i. a priori in der Anschauung darlegen, Qualitäten aber lassen sich in keiner anderen als empirischen Anschauung darstellen. Daher kann eine Vernunfterkenntnis derselben nur durch Begriffe möglich sein) (A714f/B742f).

[17] I simply note this priority of method over subject-matter here. To consider whether it is plausible would go beyond the scope of my inquiry.

concrete magnitude, for example a line, or a plane figure.[18] The term ‘quantum’ thus denotes an ontological kind: Magnitude in this sense is not something that an object *has,* but rather something that an object *is* – as a consequence of which the term ‘quantum’ admits of the plural. Thus, Kant is happy to talk about *quanta.* By contrast, ‘quantitas’ denotes the notion of magnitude with which we are familiar: it is the magnitude that an object has, its size. Thus, the *quantitas* of something is that which is given in response to the question ‘How big?’ or ‘How many?’.[19]

Like his conception of mathematics as a whole, Kant’s notion of magnitude is informed by the Euclidean tradition in mathematics, in particular by the Eudoxean theory of proportions.[20] Without entering into the details of this theory, we can simply note two central features of it that also play an important role in Kant’s conception of mathematics. First, magnitudes can be composed. For instance, two *quanta* can be added to one another so as to yield more of the same. Second, magnitudes are capable of standing in comparative size-relations. That is, one magnitude can be larger, equal to, or smaller than another. Both of these features require that magnitudes exhibit a part-whole structure. Thus, for the composition of *quanta* to be possible it must be possible to say, e.g., that two *quanta* are parts of a whole that is larger than either of them. Likewise, to say that one magnitude is greater than another is to say, on this view, that the second is equal to a proper part of the first.[21]

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[18] For this and the following see Sutherland, “Kant’s Philosophy of Mathematics and the Greek Mathematical Tradition,” as well as Sutherland, “The Role of Magnitude in Kant’s Critical Philosophy”.
[19] Cf. the following explanation from the Lectures on Metaphysics: “That determination of a thing through which one cognizes something as a quantum is quantity or magnitude” (Diejenige Bestimmung […] eines Dinges, durch welche man eine Sache als ein Quantum erkennt, ist Quantität oder Größe) (Metaphysik K3, *Ak.* XXIX, 991).
[20] This point is emphasized in Sutherland, “Kant’s Philosophy of Mathematics and the Greek Mathematical Tradition.”
[21] Compare the fourth and fifth of Euclid’s Common Notions.

Since for Kant, as for the entire tradition going back to Euclid, geometry serves as the paradigmatic sub-field of mathematics, this theory of magnitudes is meant to apply, in the first instance, to spatial magnitudes. As is emphasized by Shabel, one consequence of this is that it must be possible to cognize part-whole relations among magnitudes diagrammatically.[22] That is, it must be possible to read off from a mathematical diagram the comparative size-relations and to carry out compositional operations by means of diagrammatic representations, where, importantly, these operations must not be based on measurement. I will discuss the role of the diagram in Kant's conception of mathematics shortly.

2.2 Quantity and Strict Logical Homogeneity

As Sutherland has pointed out, Kant's theory of concepts does not allow the kinds part-whole relations that are required for the representation of *quanta* to be represented by purely conceptual means. [23] To see this recall that Kant thinks of the intension of a concept as a collection of marks. This conception is based on the Aristotelian theory of definition in terms of nearest genus and specific difference, as illustrated by Porphyrian genus-species trees. We can abstract from the details of this theory and leave out of consideration, e.g., the distinction between *essentialia*, *propria*, and accidental properties. What matters is that the theory assigns to a conceptual mark one of two roles: either the mark denotes a genus, or it indicates a specific difference.[24] Thus, if I take the concept 'animal' and treat it as a genus, I may distinguish two

[22] Cf. Shabel, *Mathematics in Kant's Critical Philosophy,* 11-21.
[23] The following discussion is indebted to Sutherland, "Kant's Philosophy of Mathematics and the Greek Mathematical Tradition," and "Arithmetic from Kant to Frege: Numbers, Pure Units, and the Limits of Conceptual Representation."
[24] These roles are, of course, relative. If A is contained under B, and B under C, then B is a species relative to C, which is its genus. But B is a genus relative to A, which is a species of it.

species by introducing ‘rational’ as a mark representing a specific difference. So the genus ‘animal’ is divided into the two species ‘rational animal’ and ‘non-rational animal’. In each case, a mark is added to the intension of my original concept to form the new species-concept. I can continue this process and distinguish further species by, say, dividing my concept ‘rational animal’ into scholars and non-scholars, and so on.[25] In each case, I effect the division of a genus into species by adding to its intension a mark which sorts the members of the genus into those that exhibit the relevant characteristic and those that do not.

We have to ask how, in such a system, one represents mere numerical difference. That is, how does one represent a species that contains under it a plurality of members which do not differ from one another by belonging to different species? It appears that one could only represent numerical difference by way of adding marks. And this means that one could represent numerical difference only by means of introducing further qualitative differences; that is, by introducing further specific differences. So the answer to the question is that in such a system it is impossible to represent numerical difference without specific difference.[26] To see this, consider the following example.

I begin with the concept ‘scholar’, and the task is to represent a plurality of scholars. Since the only means at my disposal is the introduction of conceptual marks I can represent, say, two scholars by introducing a mark that sorts scholars into short and tall. But this means that I have succeeded in representing numerical difference only at the cost of introducing further qualitative differences. It is not hard to see that this would be the case no matter how many

[25] The division need not be dichotomous, but can comprise more members, as long as there is no overlap between them and they jointly exhaust the extension of the divided concept. The dichotomous case is simply the easiest for illustration purposes. For discussion of Kant’s conception of the logical division of a concept see Anderson, “The Wolffian Paradigm,” and Wolff, *Die Vollständigkeit der kantischen Urteilstafel*, 160-170.
[26] This claim holds true even if we do admit accidental properties, as the example that follows will show.

additional marks I introduce. As long as the only means at my disposal is representation through marks, I am always limited to representing numerical difference by way of introducing further qualitative differences.

Following Sutherland, I will call any entities that differ from one another only numerically, but not qualitatively, strictly logically homogeneous.[27] As my example ought to have shown, the theory of concepts Kant espouses is incapable of representing strict logical homogeneity. That is, it is impossible, on this theory, to represent by purely conceptual means numerical diversity in the absence of qualitative (specific) diversity. Consequently, a theory of cognition according to which the only kind of representation there is are concepts would be committed to the Identity of Indiscernibles: for any objects x and y, if, for every property Φ, Φx and Φy, then x=y.[28]

Now, the composition requirement on magnitudes, according to which it must be possible to combine two magnitudes so as to have *more of the same*, requires strict logical homogeneity.[29]

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[27] Compare the following remark of Kant's from his Lectures on Metaphysics, *Ak.* XXVIII, 504: "Homogeneity is specific identity with numerical diversity, and a quantum consists of homogeneous parts".

[28] Kant famously criticizes Leibniz for his embrace of the Identity of Indiscernibles as a metaphysical principle (see A271f/B328f). We can now see why Kant sees the root of Leibniz's mistake as lying in the latter's failure to appreciate the heterogeneity of sensibility and understanding (see the passage from A271/B327 quoted at the outset of the Introduction). Since Kant shares Leibniz's conception of concepts, it is only by recognizing a distinct kind of representation from concepts, viz. intuitions, that Kant has the resources for rejecting the Principle of the Identity of Indiscernibles.

[29] It might be objected that *strict* logical homogeneity is too strong a requirement; that the composition requirement demands only that there is *some* concept under which all of the composed items fall. But this is false. To see this, consider an example. I may perform an operation of composition by adding an orange to an apple. As a result I seem to have a whole that is greater than any of its parts, for I have two pieces of fruit instead of one. Notice, however, that I do not have *more of the same*. For I do not have more apples. And while it is true that I have more pieces of fruit, my entitlement to the representation of numerical difference under the concept 'piece of fruit' rests on the qualitative distinction between apples and oranges. But the availability of qualitative differences cannot be a condition on the applicability of mathematical concepts, e.g., the natural numbers. This would infringe on the generality of the numbers, that is, the fact that anything whatsoever can be counted. It must be possible in principle to count things that do not exhibit any qualitative differences. And this means that the mathematical concept of magnitude is the concept of mere quantitative difference, that is, quantitative difference paired with strict logical homogeneity. See Sutherland, "Kant's Philosophy of Mathematics and the Greek Mathematical Tradition," 199f, for discussion.

The possibility of representing magnitudes therefore hinges on the possibility of representing numerical difference in the absence of qualitative difference. As we have just seen, for Kant this is impossible by purely conceptual means. It follows from this that mathematics is dependent on a mode of representation other than concepts. This is, of course, intuition. And we are now in a position to appreciate one of the main reasons why Kant thinks that mathematical cognition requires intuition.[30] But how does intuition succeed in representing the strict logical homogeneity that is constitutive of magnitudes?

To answer this question, consider some of the characteristics Kant ascribes to space, the pure form of outer intuition, in the Transcendental Aesthetic. Both of the following passages (which are taken from what are known as, respectively, the third and fourth Space Arguments of the Metaphysical Exposition of space) are intended to support the claim that the original representation of space is an intuition, not a concept.[31] Here is, first, the opening of the third argument:

> Space is not a discursive or, as we say, general concept of relations of things in general, but a pure intuition. For, in the first place, we can represent to ourselves only a single space; and if we speak of diverse spaces, we mean by that only parts of one and the same unique space. (A24f/B39)[32]

Next, consider the fourth Space Argument:

[30] The other main reason is that intuition serves to confer objective validity on mathematical concepts. Carson rightly presses this point against Friedman in her "Kant on Intuition in Geometry."

[31] By saying that the *original* representation of space is an intuition Kant means the following: It is not the case that there are no spatial concepts at all. For instance, there are concepts like 'the space enclosed by three straight lines'; (or, perhaps, 'the space taken up by the solar system'). In fact, geometry contains lots of such concepts. Kant's point is rather that these concepts are concepts of *space* (and are thus non-empty) only if space can also be represented intuitionally; and if, moreover, the intuitive representation of space is a priori. I cannot enter into the argument with which Kant supports this claim. The point we are concerned with, however, is clearly part of his argument. This is the point that space is a *quantum*, and that *quanta* cannot be represented by purely conceptual means.

[32] Der Raum ist kein diskursiver, oder, wie man sagt, allgemeiner Begriff von Verhältnissen der Dinge überhaupt, sondern eine reine Anschauung. Denn erstlich kann man sich nur einen einigen Raum vorstellen, und wenn man von vielen Räumen redet, so verstehet man darunter nur Teile eines und desselben alleinigen Raumes.

> Now, every concept must be thought as a representation which is contained in an infinite number of different possible representations (as their common mark), and which therefore contains these *under itself*; but no concept, as such, can be thought as containing an infinite number of representations *within itself*. It is in this latter way, however, that space is thought; for all the parts of space coexist ad infinitum. (B39f)[33]

What I want to direct our attention to is the fact that in both passages Kant ascribes a part-whole structure to space; and that the intuitive nature of space depends, at least in part, on this structure. He says that there is a single space which consists of parts. The fact that space has parts gives the notion of numerical diversity application. Thus, Kant speaks of "diverse spaces" and "an infinite number" of parts. At the same time, these are all parts of the same whole. And although this is not made explicit in these two passages, it is clear that Kant thinks that the parts of space are all strictly logically homogeneous with one another. So we have here the representation of strict logical homogeneity, that is, qualitative identity with numerical diversity. Space thus supplies the structure of a strictly logically homogeneous manifold.

That Kant takes all parts of space to be strictly logically homogeneous and yet numerically distinct is shown by the following passage:

> The concept of a cubic foot of space, wherever and however often I think it, is in itself *completely identical*. But two cubic feet are nevertheless distinguished in space by the mere difference of their locations (*numero diversa*); these locations are conditions of the intuition wherein the object of this concept is given; they do not, however, belong to the concept but entirely to sensibility. (A282/B338, my emphasis).[34]

Again, the point is that the form of sensibility, space, supplies a structure which exhibits strict logical homogeneity. In this connection, consider also the famous example Kant uses in his

[33] Nun muß man zwar einen jeden Begriff als eine Vorstellung denken, die in einer unendlichen Menge von verschiedenen möglichen Vorstellungen (als ihr gemeinschaftliches Merkmal) enthalten ist, mithin diese *unter sich* enthält; aber kein Begriff, als ein solcher, kann so gedacht werden, als ob er eine unendliche Menge von Vorstellungen *in sich* enthielte. Gleichwohl wird der Raum so gedacht (denn alle Teile des Raumes ins Unendliche sind zugleich).

[34] Der Begriff von einem Kubikfuße Raum, ich mag mir diesen denken, wo und wie oft ich wolle, ist an sich *völlig einerlei*. Allein zwei Kubikfüße sind im Raume dennoch bloß durch ihre Örter unterschieden (*numero diversa*); diese sind Bedingungen der Anschauung, worin das Objekt dieses Begriffs gegeben wird, die nicht zum Begriffe, aber doch zur ganzen Sinnlichkeit gehören.

criticism of Leibniz's doctrine of the Identity of Indiscernibles in the Amphiboly. Let two raindrops, he says, have all the same qualities. Yet it is a sufficient reason for regarding them as numerically distinct that they occupy different locations in space. What matters for our purposes is how Kant justifies this last claim. He says:

> For one part of space, although completely similar and equal to another part, is still outside the other and for this very reason is a different part from that which abuts it to constitute a greater space. And this must hold of everything, which simultaneously occupies the different locations in space, however similar and equal it may otherwise be. (A264/B320)[35]

What Kant says here is that space is made up of parts. And while these parts may be qualitatively identical, they are numerically distinct because they are "outside" one another, that is, they are different parts of space. If this is true of all parts of space, then the fact that two objects occupy distinct locations in space is a sufficient reason for their numerical distinctness, whatever their qualitative determinations.[36]

2.3. Construction of a Concept in Intuition

We have seen that mathematics is about magnitudes, and that a *quantum* (a concrete magnitude) is a whole of strictly homogeneous parts. The part-whole structure of a *quantum* is such that parts can be composed to yield more of the same and that *quanta* can stand in comparative size-relations such that, for instance, the whole of one *quantum* is equal to the part

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[35] Denn ein Teil des Raums, ob er zwar einem andern völlig ähnlich und gleich sein mag, ist doch außer ihm, und eben dadurch ein vom ersteren verschiedener Teil, der zu ihm hinzukommt, um einen größeren Raum auszumachen, und dieses muß daher von allem, was in den mancherlei Stellen des Raums zugleich ist, gelten, so sehr es sich sonsten auch ähnlich und gleich sein mag.

[36] It is interesting to note here that in his Lectures on Metaphysics Kant distinguishes between two ways in which a whole may be composed of parts. Either the parts are homogeneous with one another; that is, they are all members of the same species and thus all instantiate the same concepts. Or the parts are heterogeneous to one another; that is, they fall under different concepts. Kant calls the former a *quantum* and the latter a *compositum* (see, e.g., Metaphysik K3, *Ak.* XXIX, 990f). Characterizing space as a *quantum* therefore implies that it is a whole made up of strictly homogeneous parts.

of another. We have also seen that the strict logical homogeneity that is required for *quanta* to exhibit these properties cannot be represented purely conceptually. Rather, intuition is required for the representation of *quanta*. Finally, we have seen that, at least in the case of space, intuition is able to fulfill this representational function because space, the pure form of intuition, itself exhibits the relevant part-whole structure.

I entered into this discussion of Kant's conception of mathematics in order to advance my argument to the effect that Kant accepts a non-discursive exercise of spontaneity in what I call sensible synthesis. The claim I wish to defend is that the geometrical construction of a concept in intuition functions as the paradigm for this kind of exercise spontaneity. That is, although not all acts of sensible synthesis are instances of geometrical construction, geometrical construction is the model by reference to which we must understand all acts of sensible synthesis.[37] For this reason, we now need to turn to the notion of construction and ask what the relation is between it and the characteristics of *quanta* just identified. As before, I will limit myself to geometry and space.

To begin with, Kant's notion of construction in intuition combines two features which at first blush seem to stand in conflict with each other. On the one hand, the process of construction yields a sensible representation, an intuition. Yet, on the other hand, this representation is said to be pure, that is, independent of affection by objects.[38] The apparent conflict arises because

[37] I support this claim in §3 below.

[38] Construction is independent of affection by objects in the sense that affection is not needed to carry out a construction. To be sure, if I construct a concept with the aid of a drawing (using straightedge and compass), there is affection (hence, empirical intuition) involved. But, first, how I construct the figure is not determined by empirical intuition. Second, and more importantly, Kant holds that the drawing is only a tool. The constructive operation itself is not dependent on it and can in principle be performed without it. See Kant's comment to this effect in a footnote in the First Introduction to the *Critique of Judgment*, *Ak.* V, 198n.

From this sense of dependence we can distinguish another sense in which construction is indeed dependent on empirical intuition. This is the sense in which for Kant constructions in pure intuition are objectively valid only sensibility is defined as the capacity to have representations in virtue of being affected. However, that an intuition resulting from construction is a priori seems to imply that it cannot depend on affection. For affection involves sensation and is empirical.[39]

Kant's strategy for avoiding this problem is to argue that sensibility has a form which can be cognized non-empirically, and that this non-empirical form can itself be represented sensibly, that is, by means of intuitions.[40] This is, of course, the doctrine of pure intuition. It is worth emphasizing that this doctrine is not adequately captured by a claim such as the following: To say that the pure form of intuition is space is to say that all empirical intuitions are necessarily spatial. For this does not capture the point, essential to Kant's doctrine, that the pure form of intuition can itself be represented sensibly, yet non-empirically. In other words, it is crucial to acknowledge Kant's commitment to the claim that there is a kind of sensible representation that is non-empirical. The pure intuitions yielded by mathematical construction are instances of this kind.

How then does the mathematician produce an intuition that is not empirical? The following passage contains a condensed statement of Kant's account, and I will use it to guide my exposition:

> To *construct* a concept, however, is to exhibit a priori the intuition corresponding to it. The construction of a concept therefore requires a *non-empirical* intuition. Accordingly, the latter must, as intuition, be a *single* object, but nonetheless, as the construction of a concept (a general representation), it must in its representation express universal validity

because pure intuition is the form of possible empirical intuitions, that is, only because we can have empirical intuitions in accordance with this form. This is the point of the Axioms of Intuition (see A162-6/B202-7).

[39] Cf. A19f/B34: "The effect of an object on the capacity for representation, insofar as we are affected by it, is *sensation*. That intuition which is related to the object through sensation is called *empirical*." (Die Wirkung eines Gegenstandes auf die Vorstellungsfähigkeit, so fern wir von demselben affiziert werden, ist *Empfindung*. Diejenige Anschauung, welche sich auf den Gegenstand durch Empfindung bezieht, heißt *empirisch*).

[40] Note the implication here: not all sensible representation is empirical for Kant. This contrasts sharply with a traditional Empiricist conception of sensible representation, according to which all representations of this kind are empirical.

for all possible intuitions, which belong under this concept. Thus I construct a triangle by exhibiting the object which corresponds to this concept either by imagination alone, in pure intuition, or in accordance therewith also on paper, in empirical intuition – in both cases completely a priori, without having borrowed the pattern from any experience. The individual figure which we draw is empirical, and yet it serves to express the concept, without impairing its universality. For in this empirical intuition we consider only the act whereby we construct the concept. To this concept many determinations of the individual figure (for instance, the magnitude of the sides and of the angles) are completely indifferent. Accordingly, in considering only the act of construction we abstract from these differences, which do not alter the concept of a triangle. (A713f/B741f)[41]

In this passage, mathematical construction is characterized by the following features: (a) To construct a concept is to generate an intuition. Qua intuition, this is a singular representation ("a *single* object"). (b) This intuition at the same time functions as a general representation: it must "express universal validity for all possible intuitions, which belong under this concept," i.e. the concept whose construction the intuition is. (c) Towards the end of the passage Kant suggests that the intuition in question functions as a general representation because in considering it we "abstract from those differences", that is, from those features that are characteristic of this particular intuition but do not hold of some of the other possible intuitions that fall under the constructed concept. He suggests, further, that this act of abstraction is achieved by means of attending only to "the act whereby we construct the concept."[42] (d) Finally, Kant claims that

[41] Einen Begriff aber *konstruieren*, heißt: die ihm korrespondierende Anschauung a priori darstellen. Zur Konstruktion eines Begriffs wird also eine *nicht empirische* Anschauung erfordert, die folglich, als Anschauung, ein *einzelnes* Objekt ist, aber nichts destoweniger als die Konstruktion eines Begriffs (einer allgemeinen Vorstellung), Allgemeingültigkeit für alle mögliche Anschauungen, die unter denselben Begriff gehören, in der Vorstellung ausdrücken muß. So konstruiere ich einen Triangel, indem ich den diesem Begriffe entsprechenden Gegenstand, entweder durch bloße Einbildung, in der reinen, oder nach derselben auch auf dem Papier, in der empirischen Anschauung, beidemal aber völlig a priori, ohne das Muster dazu aus irgend einer Erfahrung geborgt zu haben, darstelle. Die einzelne hingezeichnete Figur ist empirisch, und dient gleichwohl den Begriff, unbeschadet seiner Allgemeinheit, auszudrücken, weil bei dieser empirischen Anschauung immer nur auf die Handlung der Konstruktion des Begriffs, welchem viele Bestimmungen, z. E. der Größe, der Seiten und der Winkel, ganz gleichgültig sind, gesehen, und also von diesen Verschiedenheiten, die den Begriff des Triangels nicht verändern, abstrahiert wird.

[42] I will say more about this kind of abstraction below. But we should already note that it should not be confused with the kind of abstraction involved in concept-formation, according to the traditional abstractionist picture of concept-formation to which Kant seems to subscribe, at least with regard to empirical concepts. See *Logik*, §5f.

even an empirical intuition can be considered "completely a priori" and thus function as a pure intuition. To understand Kant's notion of construction we need to be able to explain these four features. The following account will put us in a position to do so.

It will be best to have an example to hand. I will therefore begin by reproducing the proof of Euclid's proposition I.32 (to the effect that the three angles of a triangle are equal to two right angles), which Kant describes shortly after the quoted passage. Here is, first, Kant's own description of the proof, which he provides as a means of illustrating his claim that the method of mathematics is distinct from the method of philosophy. He begins by saying that if a philosopher were asked to prove this theorem he would have to rely on conceptual analysis. But since the concept of a triangle, which is defined as a plane figure enclosed by three straight lines, does not contain the mark 'equal to two right angles', the philosopher will never succeed in providing a proof. The geometer, on the other hand, will be able to give a proof since he proceeds by a different method, which Kant describes as follows:

> Now let the geometer take up this question. He at once begins to construct a triangle. Since he knows that the sum of two right angles is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line, he extends one side of his triangle and obtains two adjacent angles, which together are equal to two right anlges. He then divides the external one of these angles by drawing a line parallel to the opposite side of the triangle, and sees that there arises an external adjacent angle which is equal to an internal angle etc. In this fashion, through a chain of inferences guided throughout by intuition, he arrives at a fully evident and universally valid solution of the problem. (A716f/B745f)[43]

[43] Allein der Geometer nehme diese Frage vor. Er fängt sofort an, einen Triangel zu konstruieren. Weil er weiß, daß zwei rechte Winkel zusammen gerade so viel austragen, als alle berührende Winkel, die aus einem Punkte auf einer geraden Linie gezogen werden können, zusammen, so verlängert er eine Seite seines Triangels, und bekommt zwei berührende Winkel, die zweien rechten zusammen gleich sind. Nun teilet er den äußeren von diesen Winkeln, indem er eine Linie mit der gegenüberstehenden Seite des Triangels parallel zieht, und sieht, daß hier ein äußerer berührender Winkel entspringe, der einem inneren gleich ist usw. Er gelangt auf solche Weise durch eine Kette von Schlüssen, immer von der Anschauung geleitet, zur völlig einleuchtenden und zugleich allgemeinen Auflösung der Frage.

The first point to note in this passage is the one Kant makes in the final sentence. The geometer proves his proposition through a series of inferences that are "guided by intuition." They are guided by intuition because the geometer reads off from his diagram – and thus "sees" – various properties and relations which figure in his inferences. Thus, he sees that, after one of the triangle's sides has been extended, there are two adjacent angles, one internal to the triangle, the other external. In this manner, he apprehends the spatial relation of being next to, or adjacent, as holding of these two angles. Furthermore, he sees that the two adjacent angles are the parts of a whole, which is also an angle, viz. the straight angle that was generated by extending the base of the triangle. Again, a spatial relation is apprehended visually, this time a part-whole relation. The apprehension of this part-whole relation allows the geometer to infer, together with the premise Kant states (that two right angles are equal to all the adjacent angles that can be drawn from a point on a straight line), that the two adjacent angles are equal to two right angles.[44]

Notice that this inference is "guided by intuition" in a fairly strong sense: it could not be drawn without relying on the apprehension of the part-whole relations exhibited by the intuition. For, as we saw in the previous section, mathematical part-whole relations exhibit strict logical homogeneity and therefore cannot be represented purely conceptually for Kant. It is only by way of intuition that they can be apprehended. Therefore, the geometer's inference could not be drawn independently of the construction in pure intuition that the diagram, when read in the proper way, helps the geometer accomplish.

Notice also that there is another inference of this sort in the proof. When the geometer divides the external angle he again apprehends, with the aid of the diagram, that the three adjacent angles thus formed are parts of the whole formed by the straight angle. This is a crucial

[44] Cf. the discussion of Kant's example in Shabel, *Mathematics in Kant's Critical Philosophy,* 96-99.

step in the proof. Having established, through additional considerations (and by relying on previously proved theorems), that each of these three angles is equal to one of the three internal angles of the triangle, the geometer can now take the final step and infer that the three internal angles of the triangle jointly are equal to two right angles. Again, however, the inference relies on the apprehension of part-whole relations exhibited in the diagram. For the geometer must apprehend that the three adjacent angles in question together form the whole that is the straight angle.[45]

This example, I hope, conveys a sense of the way in which in Kant's view geometrical reasoning depends on an appeal to intuition. I now turn to the question of what it is that allows an intuition to function as a general representation, a representation that is valid of all possible triangles and is thus a sensible representation of the concept 'triangle.' In the passage from A713f/B741f quoted above, Kant says that the geometer, in the way he uses the diagram, "abstracts from" various properties of the constructed figure and attends only to "the act whereby we construct the concept." These two features of the geometer's treatment of the constructed figure go together: To attend only to the act of construction is to abstract from various other, more determinate properties of the figure. We can spell this out as follows. To begin with, the act of construction is to be thought of as a rule-governed procedure. An act of construction is an instance of carrying out an operation defined by a general rule. The most basic rules of this kind are the postulates of Euclid's *Elements*. So, for instance, 'to draw a straight line between any two points on the plane' is a construction-rule, as is 'to describe a circle with any center and

[45] To be sure, he can rely on the premise that all angles constructed from a point on a straight line are equal to two right angles. But this premise is itself known only on the basis of apprehending the part-whole relation that obtains here: The angles drawn from the point are equal to two right angles in part because they are equal to the angle formed by the straight line. And this equality-relation is cognized only on account of the fact that the angles drawn from the point constitute the parts of the whole that is the straight angle.

distance.'[46] Further rules of this kind can be generated by combining these operations with one another and with the definitions of geometrical concepts. Thus, 'to draw straight lines between any three non-collinear points on the plane' is a construction-rule, viz. the rule for constructing a triangle.

The geometer in Kant's example applies this rule when he constructs his triangle. Notice that this rule says nothing about whether the triangle is scalene or isosceles or equilateral; nor does it specify angle-size or side-length. When a triangle is drawn, however, it is necessarily determinate with respect to all of these properties. Any drawn triangle will necessarily be either scalene or isosceles or equilateral. And it will have sides of a determinate length etc. Now, to say that the geometer attends only to the act of constructing the figure is to say that he considers the figure he has drawn only with regard to those properties which follow directly from the rule that guided his construction. And this means that he abstracts from all the more determinate properties his figure exhibits. In our example, he will attend only to those properties of the triangle which result from his having followed the general rule for constructing triangles, that is, the rule that is expressed by the words 'to draw straight lines between any three non-collinear points on the plane.' Thus, he will treat as indeterminate the size of the angles and the length of the sides. To do this is to abstract from the particular angle-size his figure exhibits.

To say that the geometer abstracts from those features of the figure which are more determinate than what is contained in his construction-rule is to say, first and foremost, that no step in his proof depends on these features. To see how this might be accomplished consider once more the discussion of the proof of proposition I.32 above. For example, when the geometer apprehends that the two adjacent angles which result from extending one side of the

[46] Euclid, *The Elements*, vol. I, 154.

triangle are equal to the angle enclosed by the straight line, this cognition does not depend on the particular size of the angles in question. It would hold for any angles obtained through this procedure. Whatever the respective sizes of the internal and external angles thus obtained, these two angles will always be two parts which jointly make up the whole that is the straight angle. That this part-whole relationship holds can thus be cognized in complete independence from consideration of the angles' sizes. To treat the constructed figure as showing this is precisely to abstract from its particular features and attend only to the act whereby it was constructed. Analogous points hold for the lengths of the sides as well as all other properties that are not a direct consequence, in this sense, of the rule according to which the figure was constructed.

Above I said that there are four features of Kant's notion of mathematical construction that we need to understand: (a) The constructed figure is a singular representation, which (b) nonetheless functions as a general representation; (c) the geometer, in considering the figure, abstracts from its particular determinations and attends only to the act of constructing it; and, finally, (d) the sensible representation thus obtained is a pure intuition. The discussion of Kant's example already contained an account of (a) through (c), at least implicitly. The constructed figure is singular because it is the content of an intuition and because it has the kind of determinacy characteristic of intuitions. This means, among other things, that it possesses the strict logical homogeneity which enables it to exhibit mathematical part-whole relations and which purely conceptual representations lack. At the same time, the figure functions as a general representation because the geometrical properties and relations the geometer cognizes in it do not depend on the particular features of the figure. Rather, they depend only on those features of the figure that it has as a result of having been constructed in accordance with a general rule, e.g., the rule for constructing triangles. In abstracting from the particular determinations of the figure and treating it only as an instance of the application of the general construction-rule, in the manner indicated, the geometer treats the figure as a general representation. (b) thus depends on (c): The individual sensible representation attains generality in virtue of being treated in a particular way; specifically, by being treated as an instance of applying a general construction-rule.

It may be worth pausing to add the following point. If what Kant means by attending only to the act of constructing the figure is that one abstracts from those of its properties which are not a direct consequence of the construction-rule, in the manner indicated, then one might wonder what constructing the figure actually adds to what is already thought in the construction-rule. If it does not add anything, it is unclear why the construction is needed in the first place. The response is that constructing the figure does add something that is not already contained in the construction-rule. But it is not easy to say precisely what this is. At a first pass, we can say that the figure does, while the construction-rule does not, exhibit certain general properties of space. These are properties that are displayed by the figure even when one abstracts from its particular features in the way the geometer does when he attends only to the act. For instance, it is not part of the content of the construction-rule for triangles (and for extending a line) that the part-whole relations among angles obtain on which the proof of proposition I.32 relies,. It is, however, part of the content of the constructed figure.

This difference between construction-rule and constructed figure is explained by the following general feature of Kant's conception of the form of intuition. To construct a figure for Kant is to determine space in a particular way. The easiest way to say what this means is perhaps to put it in terms of the Aristotelian matter-form contrast. To represent a geometrical figure is, as

Kant puts it, to represent a particular space – where ‘space’ is used in a sense that admits of the plural (there are infinitely many such spaces). Each such space is a part of the one all-encompassing space there is. That is, every representation of a region of space such as a figure is a representation of a part of a whole called ‘space’. Call this the original representation of space. Space in this latter sense, however, exists, as we might put it, as a potentiality. And every act of representing a particular space is an act of actualizing this potentiality.[47] It is, to use a different Aristotelian pair of terms, an act of giving form to a matter. The crucial point is that this matter (that is, space as a potentiality) can be formed only in some ways, but not in others.[48] What this means is that only some constructions are possible, but not others. E.g. it is not possible to construct a figure enclosed by two straight lines. In this sense the original representation of space constrains geometrical construction. We can put the point by saying that the one unique space I characterized as a potentiality is the sum-total of all possible constructions. It is obviously distinct from any actual construction. However, in carrying out an act of construction the geometer either actualizes one of these possibilities or attempts to carry out a construction that is not a possibility. Whether it is the latter or the former is left open by the construction-rule. We can therefore say that the step of going from a construction-rule to its implementation “adds” the general properties of space, in virtue of which some constructions are possible, while others are not.

[47] Compare the distinction between space as it is considered in metaphysics and space as it is considered in geometry that Kant draws in a short piece entitled “Über Kästners Abhandlungen,” *Ak.* XX, 410-423 (translated in Allison, *The Kant-Eberhard Controversy*); see also the discussion of this distinction in Friedman, “Geometry, Construction, and Intuition in Kant and His Successors.” The piece is the draft of a review of a number of works by the mathematician Abraham Gotthelf Kästner. Kant never published the review under his own name, but gave it to his friend Johann Schultz, under whose name it was published in connection with the controversy between Kant and Johann August Eberhard.

[48] It is, in other words, a potentiality to take on a particular range of determinations. See the discussion of Kant’s Aristotelian conception of capacities in §1 of the Introduction.

I have not yet said why the intuition that results from constructing a mathematical concept is pure rather than empirical. But I think we are now in a position to answer this question. In the passage from A713f/B741f quoted above Kant says that the drawn figure may be an empirical intuition, but that the geometer treats it as a pure intuition – that he uses it to give an *a priori* exhibition of the geometrical concept.[49] He treats it as a pure intuition precisely because, and insofar as, he abstracts from its particular features and attends only to the act whereby it is constructed. This means that he considers only the purely spatial features of the intuition. He abstracts from anything that belongs to sensation. Thus, he abstracts from, say, the color of the figure. Since sensation is what makes an intuition empirical, an intuition that does not, in the relevant respect, involve sensation is not empirical.

§2.4. Construction, Spontaneity, and Synthesis

We now have, at least in outline, an account of Kant's conception of geometrical construction and the role it plays in mathematical proof. And we have seen that to construct a geometrical figure is to entertain a spatial representation, whether in imagination alone or with the aid of an actual physical drawing. Kant holds that such an act of spatial representation involves a spontaneous synthesis, and this claim will play a crucial role in the remainder of this chapter. For this reason, I now want briefly to consider this claim and provide textual support for it.

In constructing a geometrical figure one entertains a particular spatial representation. One represents what Kant sometimes calls a determinate space. This is done by means of a rule-

[49] Strictly speaking, the terms 'a priori' and 'pure' are not equivalent for Kant. However, since the difference between them does not bear on the present topic, I shall treat them as equivalent for the purposes of this discussion. See footnote 28 in Chapter One for references.

governed procedure. One constructs the figure by following the constructive operations permitted by the Euclidean postulates, applying these operations in accordance with the definition of the figure one is constructing. The basic Euclidean operations are drawing a line (by moving a point) and describing a circle (by rotating a line around a fixed point). One constructs, for instance, a triangle by drawing three lines in such a way that they enclose a space.

It is clear that representing a figure in this manner is not an act that can be accounted for by the receptive capacity of the mind. That would require that the act can be explained by means of the capacity to have representations in virtue of being affected by objects. But it is clear that a rule-governed procedure of the kind just described is not a matter of being affected by objects. For one thing, the procedure essentially involves general representations, viz. the rules that guide the construction. And Kant is explicit that the representation of generality is an act of spontaneity rather than receptivity. In any case, according to Kant's doctrine of the two stems of cognition, a representation that is not due to receptivity must be due to spontaneity. It follows that acts of construction are acts of spontaneity.[50]

According to the conception of geometrical construction laid out in the preceding sections, a geometrical figure can be thought of as a whole of strictly logically homogeneous parts. Indeed, as the discussion of Kant's notion of magnitude has shown, this characteristic is essential to geometrical figures. In constructing a figure, therefore, one represents a whole of homogeneous parts.[51] And, we might add, one represents it *as* a whole of homogeneous parts.

[50] For this reason, Manley Thompson compares geometrical construction to intellectual intuition: "Construction in pure intuition resembles intellectual intuition, which Kant characterizes in the Aesthetic as intuition that is 'self-activity' (B68)" (Thompson, "Singular Terms and Intuitions in Kant's Epistemology," 339).

[51] Compare the definition of 'magnitude' Kant gives in the *Metaphysical Foundations of Natural Science*: "The determinate concept of a magnitude is the concept of the generation of the representation of an object by means of the composition of the homogeneous" (Der bestimmte Begriff von einer Größe ist der Begriff der Erzeugung der

That is, the act of construction is informed, at least implicitly, by an understanding of this characteristic of the figure. But this means that this act is an act of representing what Kant calls combination.[52] Since, according to the Spontaneity Thesis, combination is never given (where this means that any act of representing combination is a spontaneous act of synthesis), it follows that the act of construction is an act of synthesis.

Moreover, it is an act of specifically sensible synthesis. According to the view defended in Chapter One, a judgment for Kant is paradigmatically an act of combining two concepts. But as we saw earlier in the present chapter, strict logical homogeneity cannot be represented by means of concepts. Since space is a strictly logically homogeneous manifold it follows that the synthesis by means of which one represents a spatial figure cannot be an act of judgment.

It might be objected that this argument only shows that construction cannot be an act that involves *only* concepts. After all, the argument in §2.2 only showed that strict logical homogeneity cannot be represented by means of concepts alone. But this leaves open the possibility that strict logical homogeneity, hence space, may be represented by means of an act involving both concepts and intuitions. And there is no reason, so the objection continues, not to construe such an act as an act of judgment.

My response to this objection is twofold. First, the argument I gave in Chapter One showed that judgment is paradigmatically a combination of concepts. Admittedly, this leaves open the possibility that such an act may *also* involve sensible representations. But since the most obvious way in which a judgment may involve sensible representations is blocked by this argument, we would need an account of how we are to conceive this possibility if it is to be a

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Vorstellung eines Gegenstandes durch die Zusammensetzung des Gleichartigen) (*MAN*, Phoronomy, def. 5, note, *Ak*. IV, 489).

[52] See the discussion of the notion of combination in Chapter Three.

serious contender. The most obvious way in which an act of judgment may involve an intuition is for the intuition to be a component of judgment. But the argument of Chapter One showed that there is no good reason to think that Kant conceives of intuition as a component of judgment. Again, then, for the objection to have force against my argument we would need a different account of how it is possible for an act of judgment to involve intuitions.

Second, assuming for the sake of argument that such an account could be given, it is hard to see what the content of a judgment that is an act of construction would be. The primary function of theoretical judgments for Kant seems to be classificatory.[53] But in an act of construction, a sensible representation is generated by means of a rule-governed procedure. It is hard to see how this could be construed as an act of classification. Surely a judgment such as is expressible by the words 'This is a triangle' is not identical to the generating of a triangular spatial figure in accordance with Euclidean construction rules. I conclude that the objection is not convincing. Accordingly, Kant appears to be committed to the view that an act of construction is an act of specifically sensible synthesis.

This point is confirmed by some of the passages in which Kant talks about the representation of spatial figures. At a number of places in the *Critique,* he uses the example of representing a line and says that to represent a line one must "draw it in thought." Consider, for instance, the following passage:

> We cannot think a line without *drawing* it in thought, or a circle without *describing* it. We cannot represent the three dimensions of space without *setting* three lines at right angles to one another from the same point. (B154)[54]

[53] See the discussion in Chapter One, §3.
[54] Wir können uns keine Linie denken, ohne sie in Gedanken zu *ziehen,* keinen Zirkel denken, ohne ihn zu *beschreiben,* die drei Abmessungen des Raums gar nicht vorstellen, ohne aus demselben Punkte drei Linien senkrecht auf einander zu *setzen* [...].

Drawing a line and describing a circle are paradigm cases of geometrical construction.[55] Indeed, they are the two basic constructive operations permitted by Euclid's postulates. In each case Kant's formulation emphasizes the constructive activity.[56] And the context makes clear that this activity is an act of specifically sensible synthesis. For the passage is presented by Kant as an example of what he calls figurative synthesis. Figurative synthesis is explicitly ascribed to a capacity Kant calls the productive imagination.[57] The productive imagination is a species of the imagination in general. The latter is the capacity for "representing an object in intuition even when it is not present" (B151). That is, the imagination is a capacity for intuitive representations, representations that are not conceptual. Figurative synthesis therefore is a type of sensible synthesis. Hence, geometrical construction is, too.

3. Geometrical Construction and Empirical Intuition

I have given a fairly detailed account of Kant's conception of geometrical construction because I wish to argue that geometrical construction functions as the model for Kant's conception of sensible synthesis. If this is right, we can make progress in understanding sensible

[55] One might ask why Kant thinks that e.g. a line can only be represented by drawing it. Surely, this thought continues, I can simply picture a line, and picture all of it instantaneously, without there being anything like drawing involved. The response to this query is as follows: It may very well be that it is psychologically possible to perceive or imagine a line instantaneously. But Kant's point seems to be that to do so would be to treat the intuition of the supposed line as an empirical intuition. In so treating the representation, one would not treat it mathematically – and thus not as an instance of the mathematical concept of a line. As a consequence, one would not be entitled, on the basis of such a representation, to attribute to this object the properties that a line, considered as the object of geometry, possesses, such as the fact that it has no width. The reason is that, as we have seen, a sensible representation is considered mathematically only to the extent that in representing it one attends only to the act of construction and thus abstracts from its particular properties. As Lisa Shabel puts it, "[the] figure is, on Kant's view, a diagram of a mental act of construction and is rendered on paper for merely heuristic reasons" (Shabel, "Kant's 'Argument from Geometry'," 213).

[56] See also the following passage: "I cannot represent a line, however small, without drawing it in thought, that is, generating from a point all its parts one after another, and thereby first obtaining this intuition" (Ich kann mir keine Linie, so klein sie auch sei, vorstellen, ohne sie in Gedanken zu ziehen, d. i. von einem Punkte alle Teile nach und nach zu erzeugen, und dadurch allererst diese Anschauung zu verzeichnen) (A162f/B203).

[57] Compare the initial discussion of this capacity in the Introduction, §6.

synthesis by focusing on geometrical construction. My goal is to argue that sensible synthesis is plausibly construed as an exercise of the capacity for apperceptive synthesis. To show this I will argue that geometrical construction is plausibly construed as an exercise of the capacity for apperceptive synthesis. However, before I present the argument for this claim I would like to provide evidence for my contention that geometrical construction serves as the model for sensible synthesis in general.

I introduced sensible synthesis as the act of the mind on account of which intuitions, the representations of sensibility, exhibit the unity characteristic of objects. Intuitions can play the cognitive role Kant assigns to them (and which he characterizes as that of giving objects to the mind) only if they are subject to an act of sensible synthesis which confers on them the unity thought in the categories. In what follows, I shall present two considerations in support of the claim that sensible synthesis (so understood) is conceived by Kant on the model of geometrical construction. First, I will discuss an argument given in the Transcendental Deduction in which the notion of pure intuition plays a prominent role.[58] This will help us understand, at least in part, the philosophical motivation Kant has for thinking of sensible synthesis in this way. Second, I will quote a number of passages that directly support my thesis and thus show that it is firmly anchored in the text.

In its B-edition version, the Transcendental Deduction famously presents an argument in two steps. It is generally agreed that the second step, unlike the first, relies on considerations pertaining to the spatio-temporal form of specifically human sensibility.[59] Roughly, Kant argues that the categories are objectively valid because to so much as apprehend an object in empirical

[58] This argument was briefly discussed in §3 of the Introduction.
[59] For discussion see the references in footnote 15 of Chapter Four.

intuition one must represent it as falling under the categories. To support the latter claim Kant appeals to the pure forms of intuition, space and time, and it is this appeal to space and time that is especially relevant to our purposes.

Kant makes two claims about space and time that play a role in his argument. First, he claims that space and time function as forms of intuition. This entails that anything that is represented in empirical intuition is represented as being in space and time.[60] But this means that every empirical intuition involves spatial representation. And from this it follows, Kant argues, that the conditions governing spatial representation in general also govern empirical intuition.

Second, space not only functions as a form of intuition, it is also itself represented as an object of intuition.[61] But this means that the intuition of space is the representation of an object. It must therefore exhibit the kind of unity that is characteristic of representations of objects, viz. categorial unity. And this is indeed what Kant claims: Because space is itself represented in intuition as an object, the intuition of space exhibits categorial unity.

From these two claims Kant infers that any empirical intuition (any intuition of something as existing in space) exhibits categorial unity. Whether this inference is valid need not concern us. What matters is the claim regarding spatial representation on which it rests, specifically the claim that any act of spatial representation must exhibit categorial unity because the act of representing space itself as an object exhibits categorial unity. This claim has an important implication, which pertains directly to the thesis I am in the process of establishing.

[60] More precisely, only objects of outer sense are represented as being in space and time. The contents of inner sense are represented only as being in time, not in space. For ease of exposition I will ignore this complication in what follows and focus only on spatial representation. Nothing in the argument hangs on this.
[61] Though not an object of empirical intuition, but rather of pure intuition.

This is the implication that the synthesis responsible for empirical intuition must be of the same kind as the synthesis responsible for the representation of space as an object.

To see this, consider the following: To say that the representation of space exhibits categorial unity is to say that it exhibits combination. Since, according to the Spontaneity Thesis, any representation of combination depends on an act of synthesis, it follows that any act of spatial representation depends on an act of synthesis. Moreover, the representation of space as an object is just the representation of space that is under consideration in geometry. Call this the pure representation of space. If the unity exhibited by empirical intuition must be the same as the unity exhibited by the pure representation of space (as Kant argues that it must), it follows that the synthesis which accounts for the former must be the same (in the relevant respect) as the synthesis which accounts for the latter.

This is an important result. It forges a link between the argument of the Transcendental Deduction and the topic of geometrical construction. The reason is that the representation of space as an object is precisely the kind of representation of space that is at issue in geometry. As Kant puts it in §26 of the B-Deduction:

> Space, represented as an *object* (as is really required in geometry), contains more than the mere form of intuition, namely the *comprehension* of the manifold that is given in accordance with the form of sensibility into an *intuitive* representation […]. (B160n)[62]

What this passage says is that in geometry space is considered as an object, not merely as a form of sensibility, and that this object is itself the content of an intuitive representation. Consequently, this representation of space exhibits the unity characteristic of an intuition qua representation of an object. It appears, then, that the argument by which Kant intends to prove, in

[62] Der Raum, als *Gegenstand* vorgestellt, (wie man es wirklich in der Geometrie bedarf,) enthält mehr als bloße Form der Anschauung, nämlich *Zusammenfassung* des Mannigfaltigen, nach der Form der Sinnlichkeit gegebenen, in eine *anschauliche* Vorstellung […].

the second step of the B-Deduction, that the categories apply to every possible object of intuition involves a claim about the kind of synthesis required for the geometrical representation of space. More specifically, the claim appears to be that the synthesis responsible for the categorial unity of empirical intuition (which Kant in §26 labels the synthesis of apprehension) is of the same kind as the synthesis responsible for the unity of the representation of space that is the object of geometry.

This does not yet establish with certainty that the synthesis of apprehension is identical to, or at least a close variant of, the kind of synthesis involved in geometrical construction. But it supports the claim that the two are closely related; moreover, that they are related in such a way that the synthesis of apprehension conforms to, or must proceed in accordance with, what we might call geometrical synthesis. And this in turn provides at least initial support for my claim that geometrical construction serves as Kant's model for sensible synthesis in general.[63]

To give further support to this claim I now want to cite two passages in which Kant says explicitly that the synthesis of apprehension is of the same kind as the synthesis involved in geometrical construction. To begin with, consider the Axioms of Intuition. In this section, which is part of the chapter on the Pure Principles of the Understanding, Kant sets out to prove that the categories of quantity necessarily apply to anything that is given in intuition.[64] The categories of quantity (unity, plurality, and totality) articulate the concept of magnitude. Therefore, to demonstrate the objective validity of the categories of quantity is to show that all intuitions are magnitudes. The principle Kant sets out to prove in the Axioms, however, is slightly more

[63] I use the term 'sensible synthesis' to pick out the genus of which the synthesis of apprehension is a species. This leaves open the possibility that geometrical synthesis is also a type of sensible synthesis, yet not identical to synthesis of apprehension.
[64] For a detailed discussion of this proof see Sutherland, "The Point of Kant's Axioms of Intuition."

specific. It is the principle that all intuitions are extensive magnitudes.[65] But this complication need not concern us. What is of interest to us is that, as part of his argument for this principle, Kant draws the same inference I just discussed in connection with §26 of the Transcendental Deduction. Here is the relevant passage:

> All appearances contain, as regards their form, an intuition in space and time, which grounds all of them a priori. They cannot be apprehended, therefore, i.e., taken up into empirical consciousness, except through the synthesis of the manifold through which the representations of a determinate space or time are generated, i.e., through the composition of that which is homogeneous and the consciousness of the synthetic unity of this manifold (homogeneous). (B202f)[66]

There are three claims being made here. In the first sentence of the passage, Kant reiterates his contention that the representations of space and time function as forms of intuition, which is to say that anything apprehended in empirical intuition (that is, all appearances) is represented as being in space and time. Second, Kant claims that it follows from this that appearances can only be apprehended by means of the same kind of synthesis through which a determinate space or a determinate time are represented. Finally, this synthesis is characterized as an act of composition of a homogeneous manifold.

Of particular interest to us are the second and third claim. The apprehension of appearances of which Kant speaks here is the apprehension of objects in empirical intuition. So what is at issue is empirical intuition. Of empirical intuition Kant says that it involves the kind of synthesis through which a determinate space is represented. A determinate space contrasts with space considered as the mere form of sensibility, which we may think of as a potentiality for

[65] See A162/B202.

[66] Alle Erscheinungen enthalten, der Form nach, eine Anschauung im Raum und Zeit, welche ihnen insgesamt a priori zum Grunde liegt. Sie können also nicht anders apprehendiert, d. i. ins empirische Bewußtsein aufgenommen werden, als durch die Synthesis des Mannigfaltigen, wodurch die Vorstellung eines bestimmten Raumes oder Zeit erzeugt werden, d. i. durch die Zusammensetzung des Gleichartigen und das Bewußtsein der synthetischen Einheit dieses Mannigfaltigen (Gleichartigen).

being determined in specific ways.[67] The contrast here is the same as what Kant characterizes as the contrast between space considered as an object, on the one hand, and space considered as a mere form of intuition, on the other, in the passage from B160n quoted above. In that same passage he says that the representation of space considered as an object is the representation of space that is treated in geometry. This implies that the synthesis involved in empirical intuition is exactly the kind of synthesis that is responsible for geometrical construction.

The third claim of the passage, which characterizes this synthesis as the composition of a homogeneous manifold, confirms this point. We saw in the preceding section that Kant thinks of space as a magnitude. As a magnitude, space is characterized by strict logical homogeneity. A spatial manifold is a manifold of elements that are qualitatively identical but numerically distinct. The synthesis of a homogeneous manifold is a figurative synthesis. The drawing of a line is Kant's preferred example of this "synthesis of the manifold through which the representations of a determinate space or time are generated."

Finally, consider the following passage drawn from a discussion of the distinction between logical possibility and real possibility:

> It may look, to be sure, as if the possibility of a triangle could be cognized from its concept in itself (it is certainly independent of experience); for indeed we can give an object to it entirely a priori, i.e., we can construct it. But since this is only the form of an object, it would still always remain only a product of the imagination, the possibility of whose object would still remain doubtful, as requiring something more, namely that such a figure be thought solely under those conditions on which all objects of experience rest. Now that space is a formal a priori condition of outer experiences, that this very same formative synthesis by means of which we construct a figure in imagination is entirely identical with that which we exercise in the apprehension of an appearance in order to make a concept of experience of it – it is this alone that connects with this concept the representation of the possibility of such a thing. (A223f/B271)[68]

[67] See the remarks at the end of §2.3 above.

[68] Es hat zwar den Anschein, als wenn die Möglichkeit eines Triangels zus seinem Begriffe an sich selbst könne erkannt werden (von der Erfahrung ist er gewiß unabhängig); denn in der Tat können wir ihm gänzlich a priori einen Gegenstand geben, d. i. ihn konstruieren. Weil dieses aber nur die Form von einem Gegenstande ist, so würde er

In this passage, Kant gives a condensed version of the argument for his view that geometry is objectively valid only because its object, space, constitutes the form of empirical intuition, and thus the form of objects of experience – the only kinds of objects of which we can have knowledge. In other words, the cognitive status of geometry as a body of *synthetic* knowledge rests on the fact that, as Kant puts it in the final sentence, space is "a formal a priori condition of outer experiences." The argument as a whole need not concern us. I have quoted the passage in full mainly to provide enough context for that part of it which is of primary interest to us. This is the part towards the end of the passage in which Kant comments on the relation between the synthesis involved in constructing a geometrical figure, on the one hand, and the apprehension of appearances in empirical intuition, on the other.

The claim Kant makes here is the following: The cash value of saying that space is a formal condition of outer experiences is that the same kind of synthesis by means of which we represent space in geometry is also at work in empirical spatial representation, that is, in empirical intuition. The synthesis "by means of which we construct a figure in imagination" is identical to the synthesis "which we exercise in the apprehension of an appearance." In other words, the synthesis of apprehension that is responsible for the unity of empirical intuition is a type of figurative synthesis. It follows from this that geometrical construction serves as the model for (indeed, according to this passage, is identical to) sensible synthesis in general.[69]

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doch immer nur ein Produkt der Einbildung bleiben, von dessen Gegenstand die Möglichkeit noch zweifelhaft bliebe, als wozu noch etwas mehr erfordert wird, nämlich daß eine solche Figur unter lauter Bedingungen, auf denen alle Gegenstände der Erfahrung beruhen, gedacht sein. Daß nun der Raum eine formale Bedingung a priori von äußeren Erfahrungen ist, daß eben dieselbe bildende Synthesis, wodurch wir in der Einbildungskraft einen Triangel konstruieren, mit derjenigen gänzlich einerlei sei, welche wir in der Apprehension einer Erscheinung ausüben, um uns davon einen Erfahrungsbegriff zu machen, das ist es allein, was mit diesem Begriffe die Vorstellung von der Möglichkeit eines solchen Dinges verknüpft.

[69] Compare also the following two passages from the Axioms of Intuition:

This claim, then, is supported both by direct textual evidence and by consideration of the role that the pure intuition of space plays at a crucial juncture in the argument of the Transcendental Deduction. This ought to confer sufficient plausibility on the exegetical strategy I have been pursuing here, of turning to Kant's theory of geometrical construction in order to illuminate the notion of sensible synthesis in general, and thus to gain insight into Kant's conception of the way in which the understanding is involved in actualizations of sensibility.

4. Sensible Synthesis and Apperceptive Synthesis

I have argued that sensible synthesis (that is, the synthesis responsible for the categorial unity of an intuition) is specifically distinct from judgment. More specifically, I have argued that Kant thinks of sensible synthesis on the model of the kind of synthesis that is involved in the construction of a geometrical figure. All spatial representation, Kant seems to hold, involves an act of the mind that is paradigmatically exhibited in the pure representations of space that are employed in geometry. If this is right, then Kant is committed to the view that the spontaneous capacity of the mind (i.e., the understanding) can be exercised in two distinct ways, judgment and sensible synthesis. But this creates the problem of giving an account of spontaneity that makes such a view intelligible. How is it possible for one and the same capacity to be exercised

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Empirical intuition is possible only by means of the pure intuition of space and of time; what geometry asserts of the latter is therefore undeniably valid of the former. (Die empirische Anschauung ist nur durch die reine (des Raumes und der Zeit) möglich; was also die Geometrie von dieser sagt, gilt auch ohne Widerrede von jener […].) (A165/B206)

The synthesis of spaces and times, being a synthesis of the essential forms of all intuition, is what at the same time makes possible the apprehension of appearance, and consequently every outer experience and all cognition of the objects of such experience. Whatever mathematics in its pure use establishes in regard to the former is also necessarily valid of the latter. (Die Synthesis der Räume und Zeiten, als der wesentlichen Form aller Anschauung, ist das, was zugleich die Apprehension der Erscheinung, mithin jede äußere Erfahrung, folglich auch alle Erkenntnis der Gegenstände derselben, möglich macht, und was die Mathematik im reinen Gebrauche von jener beweiset, das gilt auch notwendig von dieser.) (A165f/B206).

in two distinct ways? More precisely, how must the capacity be characterized for such a scenario to be intelligible? This is what I have been calling the Unity Problem, the problem of giving an account of spontaneity that allows us to understand this capacity as a unified capacity, yet at the same time attribute to it two distinct acts. My proposal for solving this problem is that spontaneity (alternatively, the understanding) must be conceived, at the most fundamental level, as a capacity for apperceptive synthesis. Conceiving of the understanding in this way makes it intelligible that this capacity can be exercised both in judgment and in sensible synthesis.

For this proposal to work it has to be shown that both judgment and sensible synthesis can plausibly be construed as acts of apperceptive synthesis. I made the case that judgment can plausibly be so construed in Chapter Four. I now turn to making the parallel case for sensible synthesis.

That Kant is committed to thinking of sensible synthesis as an act of the capacity for apperceptive synthesis is implied by the Spontaneity Thesis in conjunction with his characterization of spatial representation. However, showing *that* Kant is committed to this claim does not yet help us understand how it is possible for him to hold such a view. In particular, it does not show that the characteristics he attributes to sensible synthesis make it plausible to think of this act as an act of apperceptive synthesis. Still, spelling out the argument establishing Kant's commitment will help us discharge the latter task.

The Spontaneity Thesis, which I discussed in Chapter Three, commits Kant to holding that no representation of combination can be a matter simply of receiving impressions in sensibility. As the preceding sections of the present chapter have shown, Kant regards spatial representation as being distinct in kind from conceptual representation, hence as essentially involving intuition. Part of the reason for this is the distinctive part-whole structure of space. As Kant argues in the Transcendental Aesthetic, space is such that any part of it must be conceived as a limitation of the whole of space. That is, any part of space must be conceived as part of a whole that is prior to its parts in the sense that it is not possible to conceive of something as a part of space independently of relating it to this whole. Space cannot be conceived, therefore, as a whole of independently available parts.[70]

If this kind of part-whole structure is essential to space, then any representation of space must include an at least implicit understanding of it. Otherwise it would not count as a representation of *space.* Every representation of space, therefore, must contain a consciousness of the fact that the particular space represented is part of a larger whole (of qualitatively identical parts). Such a consciousness of the relation that a particular part of space bears to other parts of space exceeds that which can be accounted for by the resources of a merely receptive capacity of representation. Rather, it involves the representation of combination, and according to the Spontaneity Thesis, any representation of combination is an act of spontaneity.

In Chapter Four I argued that an act of apperceptive synthesis is a self-conscious act of representing combination. The act is self-conscious in the sense that it presupposes a grasp, at least implicitly, of the nature of the capacity whose act it is. It is a self-conscious act of representing combination because the representation of combination is achieved by means of this grasp of the capacity's nature. I argued, further, that this grasp of the capacity's nature consists in a representation of the possible forms that combination can take. And I spelled out this idea in

[70] I quoted the relevant passages from the Aesthetic above in §2.2. See also the following passage from the Second Antinomy: "Properly speaking, one should call space not a *compositum* but a *totum*, because its parts are possible only in the whole, and not the whole through the parts" (Den Raum sollte man eigentlich nicht Compositum, sondern Totum nennen, weil die Teile desselben nur im Ganzen und nicht das Ganze durch die Teile möglich ist) (A438/B466).

terms of the notion of a mode of combination. Modes of combination come in two kinds, in accordance with the two kinds of exercise of which the capacity for apperceptive synthesis is capable. The logical forms of judgment constitute the modes of combination constitutive of the exercise of the capacity for apperceptive synthesis in judgment. By contrast, the schematized categories, I claimed, constitute the modes of combination constitutive of the exercise of this capacity in sensible synthesis. What we now need to do is to spell out this last claim.

I said above that for Kant spatial representation involves a consciousness of the homogeneous nature of space, as well as a consciousness of the relation that a given part of space bears to other parts of space. Kant accounts for this consciousness by means of the notion of a schema. The doctrine of the schematism to which this notion belongs is complex and subject to controversy. Considering it in detail would take us too far afield. But by staying close to Kant's text we can isolate some of the crucial features of this doctrine, and that is all we need for our purposes.

Since the discussion so far has focused on geometrical representation I will concentrate on the categories of quantity and the associated schemata, which are central to geometrical representation. A full account of sensible synthesis would need to include a treatment of the remaining categories (and schemata). It would also need to take into account the relation between spatial representation and temporal representation. Again, this would go beyond the confines of this chapter. But since geometrical construction serves as the model for sensible synthesis, it is clear that an account of how the schematized categories function as modes of combination in sensible synthesis must begin with the categories of quantity.[71]

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[71] In the *First Critique*, Kant specifies the schemata for the categories in terms of time, the form of inner sense. Since all appearances are in time, but only those of outer sense are in time and space, this is the proper way to

Consider once more a passage from the Axioms of Intuition that I already discussed in part in §3 above:

> All appearances contain, as regards their form, an intuition in space and time, which grounds all of them a priori. They cannot be apprehended, therefore, i.e., taken up into empirical consciousness, except through the synthesis of the manifold through which the representations of a determinate space or time are generated, i.e., through the composition of that which is homogeneous and the consciousness of the synthetic unity of this manifold (homogeneous). Now the consciousness of the homogeneous manifold in intuition in general, insofar as through it the representation of an object first becomes possible, is the concept of a magnitude (*quantum*). (B202f)

As I said in §3, the claim advanced in this passage is that the synthesis of apprehension is identical to the synthesis involved in geometrical construction. What I want to focus on now is how Kant characterizes this synthesis here. He says, first, that this synthesis includes a "consciousness of the synthetic unity of [a] manifold [of homogeneous elements]." This formulation echoes the claims Kant makes about the synthetic unity of apperception in §16 of the B-Deduction, which I discussed in Chapter Four. In accordance with the interpretation developed there, I take this passage to be saying that geometrical synthesis depends on a consciousness of the mode of combination that constitutes the form of the representation of combination which this act accounts for.[72] Kant then goes on, in the final sentence of the passage, to identify this

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proceed, given the purposes of the *Critique*. However, it must also be possible to relate the categories to the pure form of outer sense, space, and thus to do for space what the chapter on the Pure Principles in the *Critique* does for time, viz. to specify in terms of the form of intuition what it is for an object in general to be given in intuition. Kant does this in the *Metaphysical Foundations of Natural Science,* whose topic is the concept of an object of outer sense in general (which, according to Kant, is the concept of matter). Accordingly, it is here that we find an account of what it is for an object to be given not just in time, but in time and space.

[72] Compare the following passage from §17 of the B-Deduction: "But in order to cognize something in space, e.g., a line, I must *draw* it, and thus synthetically bring about a determinate combination of the given manifold, so that the unity of this act is at the same time the unity of consciousness (in the concept of a line); and it is through this that an object (a determinate space) is first cognized." (Um aber irgend etwas im Raume zu erkennen, z. B. eine Linie, muß ich sie *ziehen*, und also eine bestimmte Verbindung des gegebenen Mannigfaltigen synthetisch zu Stande bringen, so, daß die Einheit dieser Handlung zugleich die Einheit des Bewußtseins (im Begriffe einer Linie) ist, und dadurch allererst ein Objekt (ein bestimmter Raum) erkannt wird) (B137f).

consciousness as the concept of magnitude. And the concept of magnitude is equivalent to the three categories of quantity taken together.[73]

Now, the schema associated with the categories of quantity is number. But this schema relates specifically to time, the form of inner sense. Since we are concerned with space, the form of outer sense, we cannot directly apply to our case what Kant says about the schema of quantity.[74] However, it is clear that the general characterization he gives of this schema applies to what we might call the spatial (or spatio-temporal) schematization of quantity as well. He says of the representation of number that it is "nothing other than the unity of the synthesis of the manifold of a homogeneous intuition in general" (A143/B182). Since space is a homogeneous manifold just as much as time, the characterization applies to the case of space as well.

Notice that the formulation Kant uses here echoes the characterization he gives of the pure concepts of the understanding in the Metaphysical Deduction (which I discussed in Chapter Two, §3.3), with the added specification that the manifold of intuition is a homogeneous manifold, that is, a manifold of strictly logically homogeneous elements. According to the

[73] A magnitude is a manifold (plurality) of homogeneous parts (unity) considered as forming a whole (totality). Accordingly, Kant sometimes speaks of the concept of magnitude as a pure concept of the understanding, that is, a category; see, e.g., A142/B182. Consider also the following passage from one of the Lectures on Metaphysics: "The category of magnitude, for instance, as a manifold of homogeneous parts that together constitute a one – this cannot be comprehended apart from space and time" (Z. E. die Categorie der Größe, als ein Vieles gleichartiges, so zusammen Eines ausmacht: diese läßt sich ohne Raum und Zeit nicht begreifen) (Metaphysik Arnoldt (K3), 1794/95, *Ak*. XXIX, 979).

[74] Although strictly speaking the *Critique* recognizes schematized categories only in relation to the form of inner sense, time, in the Axioms of Intuition Kant actually comments briefly on the parallel case of space:

> On [the] successive synthesis of the productive imagination, in the generation of shapes, rests the mathematics of extension (geometry) with its axioms, which express the conditions of sensible intuition a priori, under which alone *the schema of a pure concept of outer appearance* can come about, e.g., between two points only one straight line is possible; two straight lines do not enclose a space, etc. (Auf [die] sukzessive Synthesis der produktiven Einbildungskraft, in der Erzeugung der Gestalten, gründet sich die Mathematik der Ausdehnung (Geometrie) mit ihren Axiomen, welche die Bedingungen der sinnlichen Anschauung a priori ausdrücken, unter denen allein das Schema eines reinen Begriffs der äußeren Erscheinung zu Stande kommen kann; z. E. zwischen zwei Punkten ist nur eine gerade Linie möglich; zwei gerade Linien schließen keinen Raum ein etc.) (A163/B204, my emphasis).

interpretation I have developed, the schematized categories function as sensible modes of combination, that is, as modes of combination which guide the act of sensible synthesis. Being guided in this manner makes the act of sensible synthesis an act of apperceptive synthesis. For what it means to be guided in this manner is that the act of synthesis is effected by means of a consciousness of the mode of combination that the act takes. Accordingly, Kant's characterization of the schematized category of quantity as a representation of the unity of the synthesis of a homogeneous manifold in general supports my contention that sensible synthesis (that is, the synthesis of manifolds that exhibit strict logical homogeneity) is a species of apperceptive synthesis.

In Chapter Four I argued that, as modes of concept-combination, the logical forms of judgment function as representations of the unity of discursive synthesis, that is, judgment. The act of judgment depends on an at least implicit consciousness that the unity that concepts are represented as having in a judgment is a specific instance of a general form which can be instantiated in other cases as well, and in virtue of exhibiting which the given judgment is related to other possible judgments. We are now in a position to see that the concept of magnitude plays an analogous role in the case of sensible synthesis. One generates a geometrical figure by means of a consciousness of the relevant construction rules. The construction rules, which constitute what we might call the spatial schema of the category of magnitude, function as the modes of combination for sensible synthesis. It is only by means of a consciousness of these rules that an act of figurative synthesis is an act of construction. Indeed, as we saw in §2.3 above, it is essential to acts of construction that "we attend only to the act whereby we construct the concept" (A714/B742). But this means that what we must attend to is the sensible mode of combination. It follows that acts of construction constitutively depend on a consciousness of the modes of combination they employ. This makes geometrical synthesis self-conscious in the sense discussed in Chapter Four. Accordingly, geometrical synthesis is a species of apperceptive synthesis.[75]

Since Kant holds that all intuitions of outer sense depend on a synthesis that is either identical to or modeled on geometrical synthesis, it follows that sensible synthesis in general is a species of apperceptive synthesis. Although it is clear that Kant is committed to this claim, this does not yet show us how to apply the account of geometrical synthesis I have developed to sensible synthesis in general. This would require an account of how the principles governing geometrical synthesis, which concern only the categories of quantity, must be supplemented so as to yield a spatial schematization of the remaining categories. This is a substantive task, which would have to take into account Kant's discussion of the concept of matter in the *Metaphysical Foundations of Natural Science.* However, our present purpose does not require us to confront this task. The goal here was only to show that sensible synthesis is an act of the capacity for apperceptive synthesis. Showing this requires us only to establish that specifically geometrical synthesis is an act of apperceptive synthesis. And this has now been accomplished.

In this chapter have argued that consideration of Kant's conception of geometrical construction shows that Kant recgonizes an exercise of spontaneity in sensible synthesis which is distinct from judgment. In constructing a concept in intuition (which is what one does in

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[75] Compare Friedman's comment that "geometrical motion in this sense [viz., drawing a line by moving a point, describing a circle by rotating a line; that is, the two fundamental Euclidean constructive operations] is a direct expression of the transcendental unity of apperception" (Friedman, "Geometry, Construction, and Intuition in Kant and His Successors," 198). Although Friedman appreciates the distinctively sensible nature of geometrical construction in Kant, he does not help us understand *how* the act of construction can be an expression of the transcendental unity of apperception; nor does he provide an account of the relation between construction, apperception, and judgment.

geometry, according to Kant) one generates a spatial representation. This representation must contain in its content the relevant essential characteristics of space. One such characteristic is that space exhibits strict logical homogeneity; that is, space is a manifold of qualitatively identical elements. However, strict logical homogeneity cannot be represented by purely conceptual means. Spatial representation, therefore, is sensible; it essentially involves intuition. In this regard, spatial representation differs from judgment, which does not involve intuition in this way.[76] At the same time, any act of spatial representation requires an act of synthesis. Accordingly, this act of synthesis is of a kind that is distinct from judgment.

Although sensible synthesis is distinct in kind from judgment, like judgment it can be understood as an act of apperceptive synthesis. Like judgment, sensible synthesis is an act of representing combination which depends on a consciousness of its own form; more precisely, on a consciousness of a sensible mode of combination. But since sensible modes of combination are distinct from modes of concept-combination, sensible synthesis constitutes a distinct kind of exercise of the capacity for apperceptive synthesis from judgment. Judgment and sensible synthesis, then, are generically identical but specifically distinct. That is, judgment and sensible synthesis constitute two different acts of a single capacity, which is properly characterized as the capacity for apperceptive synthesis.

By showing that, like judgment, sensible synthesis can be understood as an act of apperceptive synthesis I have now provided a solution to the Unity Problem. The Unity Problem required us to make it intelligible that two acts that are distinct in kind (judgment and sensible synthesis) can both be attributed to the same capacity (the understanding). By demonstrating that

[76] To be sure, judgment depends on intuition for its objective validity. But this is a different kind of relation to intuition. The act of judging does not itself involve (at least not constitutively) an act of intuiting. By contrast, an act of spatial representating does constitutively involve an act of intuiting.

both judgment and sensible synthesis can be comprehended as acts of apperceptive synthesis we have met this requirement. As a result, we can now comprehend, at least in outline, how it is so much as possible for Kant coherently to maintain that understanding and sensibility are heterogeneous capacities, yet at the same time argue that the understanding is itself involved in the actualization of sensibility. Accordingly, we are now in a position to appreciate the full significance of Kant's remark, already quoted several times, that "[it] is one and the same spontaneity, which in the one case, under the title of imagination, and in the other case, under the title of understanding, brings combination into the manifold of intuition" (B162n).